A New Paradigm For Modeling The Precision Strike Process (U)

Henry Neimeier

The MITRE Corporation
1820 Dolley Madison Boulevard
McLean, Virginia.

(U) Abstract

(U) In this paper we describe a new paradigm for modeling, and apply it to a simple view of the precision strike attack process against mobile targets. The new modeling paradigm employs analytic approximation techniques that allow rapid model development and execution. These also provide a simple dynamic analytic risk evaluation capability for the first time. The beta distribution is used to summarize a broad range of target dwell and execution time scenarios in compact form. The data processing and command and control processes are modeled as analytic queues.

(U) Two versions of the model are presented. The first static model calculates lethality for direct and commander-in-the-loop paths. A secondary throughput measure gives the proportion of sensor messages that reach the executing weapon platform. The impact of variability in workload and service time is assessed. A fractile technique is used to encode uncertainty in sensor workload and process service time. The second dynamic model considers a time varying workload and service time distribution. Lethality, total sensor to target delay probability, and throughput probability are calculated as a function of time. This paper outlines the techniques, more details are provided in references 1-4.

(U) Model Of Detection-Processing-Attack

(U) A key question with mobile targets is whether the weapon can reach the target before it moves. Figure 1 presents a simple model of the precision strike attack process. Sensor systems (e.g., SIGINT, IMINT) scan the environment for potential mobile targets. On sensor input, a message with the appropriate data is passed on for processing. These messages form the workload for the data processing and command and control queues. The data processing and command and control service times for these messages are obtained from empirical data. Variability in service time is specified by the coefficient of variation (standard deviation in service time divided by mean service time). Higher workloads or more variable service times result in longer the queue processing delays. On completion of data processing, there are two potential paths to the attacking weapon system: direct or via command and control headquarters. In the command and control path, the processed message data is passed to a command facility which then passes the attack order to the weapon system. This has the advantage of threat assessment and optimal weapon allocation with consequent lower attack losses. The main disadvantage is the added processing time. The attacking weapon system takes time to reach the target (Weapon Delay node). Target dwell time is a function of the target type, operating doctrine, and relative time when the target is detected (Target Dwell Minutes node). Spare minutes is calculated by subtracting processing and weapon delay from target dwell minutes (Spare Minutes node). Lethality is the proportion of time the weapon system reaches the target before it moves (postive spare minutes). Throughput is the proportion of messages that can be processed within the system capacity limitation.

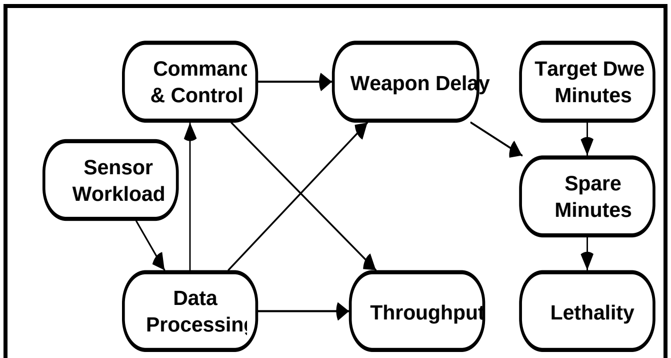

Figure 1. (U) Simple Precision Strike Process Model Unclassified

(U) New Simulation Paradigm

(U) A new simulation paradigm is proposed to overcome several of the limitations of discrete event simulation. It is based on the combination of analytic queuing networks and analytic uncertainty modeling. The analytic queue technique gives an approximate transient solution to the general inter-arrival time and general service time single server queue. Analytic uncertainty analysis is based on the beta distribution. It provides the entire uncertainty probability distribution vice an uncertain estimate of the mean. The beta distribution can be fit based on the minimum, mean, maximum, and standard deviation statistics. In a complex uncertainty calculation all that is required is to keep track of these statistics as the calculation proceeds. At any point in a calculation, the probability distribution of the result can be derived by fitting a beta distribution based on these four statistics. When analytic queuing is combined with analytic uncertainty, modeling dynamic uncertainty analysis becomes feasible. The time varying uncertainty distribution in resulting measures of effectiveness can be calculated at any specified time or over any user specified time interval. This new capability is not available in discrete event simulation.

(U) Simulations can be viewed as a network of interrelated queues. Thus the new paradigm has wide applicability. Its deterministic solution greatly simplifies sensitivity and uncertainty analysis of complex, many-parameter models. The factor effects in a many factor model are very difficult to obtain in discrete event simulation since they are masked by the stochastic simulation uncertainty. In discrete event simulation the causal chain between input parameter change and resulting output measure effect is broken. Due to the stochastic random number generation process, many runs could be required to see the effect of an input parameter change. In training simulations this random learning effect can be a problem. Our proposed deterministic technique overcomes this problem.

(U) Discrete event simulation requires a time period of many simulation events to determine sample statistics. Our technique provides instantaneous statistics.

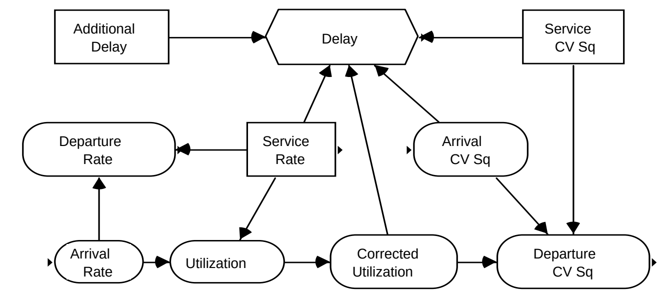

Figure 2. (U) Analytic Queue Node

Unclassified

(U) Analytic Queuing

(U) Figure 2 shows an analytic queuing node in the DEMOS™ language (see references 5-6). It is duplicated for all process components. In our case it is used to model the data processing and command and control nodes. The calculated output is process delay (hexagon). Each icon has an associated equation. The delay equation in DEMOS™ is (see source reference 3):

1) Delay=(((((Uc*(Ca+Cs))/(2*(1-Uc)))+1) /Sr)+Ad)
where:

Uc is the corrected process utilization
Ca is the square coefficient of variation
in inter-arrival times of messages
Cs is the square coefficient of variation in
process service time
Sr is the service rate for the process,
or the reciprocal of the service time
Ad is additional process delay

(U) Inputs to the process are in rectangles. These include additional delay, service rate, and service coefficient of variation squared. Additional delay is the fixed component of processing time. It might represent a fixed formatting or communication time per message. Service time is the processing time per message. The variability in service time is specified by the service coefficient of variation squared (Cs). It is defined as the square of the standard deviation in service time divided by the mean service time. Cs is unity for an exponential process, greater than one for a hyperexponential, and zero for a fixed service time. Ca measures the variability in message inputs. It is specified along with the arrival rate of messages in the “Sensor Workload” block of figure 1. Utilization is the arrival rate divided by the service rate. In this simple model we use a corrected utilization:

2) Uc = if u<.8 then u else if u>1 then .9
else .8+(u-.8)/2

Thus the maximum utilization is set at .9. In this simple model messages are lost when message arrival rate exceeds service rate. The departure rate from the process node is the minimum of the arrival or service rate.

3) Departure Rate = Min([Ar,Sr]))

Finally the message departure coefficient of variation squared is calculated from (see source reference 3):

4) Departure CV Sq = (((Uc*Uc)*Cs)
+((1-(Uc*Uc))*Ca))

At high utilization the service time coefficient predominates while at low utilization the arrival coefficient dominates. Note that the departure rate from this process queue serves as the arrival rate to the next queue along the process chain. Similarly the departure coefficient of variation squared serves as the arrival coefficient of variation squared for the next queue along the process chain.

(U) Beta Distribution Method

(U) The beta distribution method is useful whenever sensitivity analysis is important. It provides the entire resulting probability distribution vice a single uncertain point estimate of the mean. Both analytic development costs, and computer execution costs are far less than in discrete event simulation. The price paid is some lack in modeling flexibility.

(U) With the appropriate choice of parameter values, the beta distribution closely fits all the classical probability distributions. The sums and products of beta variates are also approximately beta distributed. The beta distribution can be fit based on the minimum, mean, maximum, and standard deviation statistics. In a complex calculation, all that is required is to keep track of these statistics as the calculation proceeds. At any point in a calculation, the probability distribution of the result can be derived by fitting a beta distribution based on the four statistics.

(U) The beta distribution can be fit based on the minimum, maximum, mean, and standard deviation. The fit parameters a and b are based on a range variable r and a skewness variable s, and are defined in standard terms as (see source reference 4):

5) r = (standard deviation / (maximum - minimum))^2
6) s = (mean - minimum) / (maximum- minimum)
7) a = s^2 (1 - s) / r - s
8) b = s (1 - s) / r - 1 - a

(U) Converting From Queue Statistics To Beta Statistics

(U) The static model includes uncertainty in sensor workload and uncertainly in mean service rate. Uncertainty is encoded by entering values for three fractile probabilities (1/6, 1/2, and 5/6). Each fractile represents the cell midpoint of a third of the probability. The sensor workload arrival rate fractiles are 1.2, 1.6, and 2.0 messages per minute. The service rate fractiles for the data processing and command and control queues are both the same at 1.25, 1.75, and 2.25 messages per minute. Now all queue calculations that use both arrival rate and service rate have nine result values (3 X 3 matrix). The mean and standard deviation of these nine values are used for the total delay beta mean and standard deviation statistics. Each of the 9 matrix values represent mid-points of 1/9 of the total probability space. To calculate the minimum beta value we subtract 1/18 of the difference between the maximum and minimum value from the minimum value. This approximates the distance between the cell mid-point and its starting value. Similarly we add 1/18 of the minimum maximum range to the largest value to determine the maximum beta value.

(U) Weapon Delay and Target Dwell Minutes

(U) The weapon delay distribution summarizes a broad range of attack platforms and target location scenarios. The weapon delay is the distance to the target divided by the platform speed. The
mean weapon to target transit time is set to 10 minutes with a standard deviation of 2 minutes. The minimum transit time is 2 minutes and the maximum transit time is 15 minutes. This results in a beta distribution with a negative skew (see figure 3).

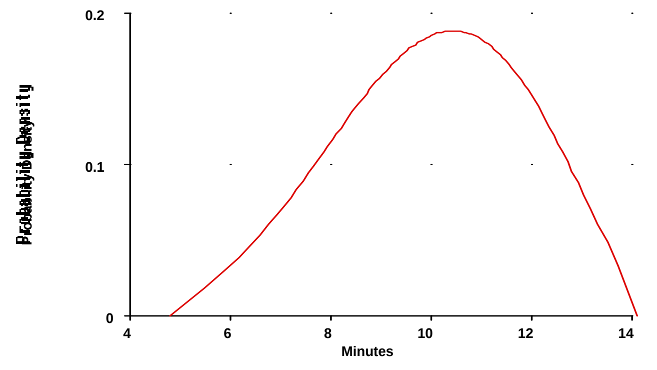

Figure 3. (U) Weapon Delay Distribution
Unclassified

(U) The target dwell time distribution summarizes a broad range of mobile target types, operating doctrine, and environmental conditions. Mean target dwell time is set to 15 minutes with a standard deviation of 5 minutes. The minimum dwell time is five minutes and the maximum dwell time is 45 minutes. This results in a beta distribution with a positive skew (see figure 4). This target class might represent mobile ballistic missiles or netted mobile air defense radars. Other target classes can be represented by changing the input beta statistics. For example, tanks or isolated air defense radars would have higher dwell time statistics.

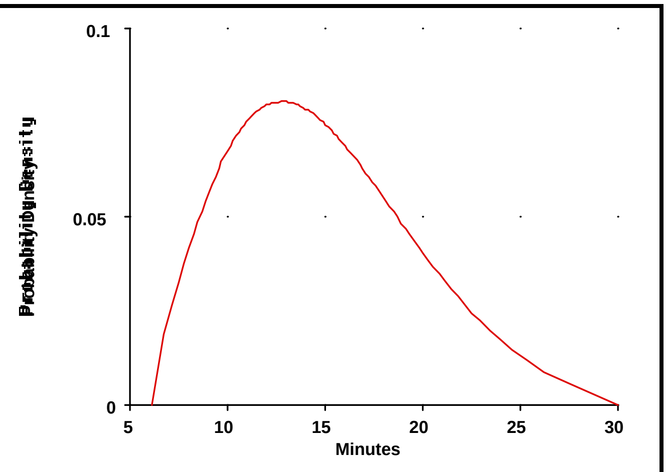

Figure 4. (U) Target Dwell Distribution
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(U) Lethality Calculation

(U) The total time for the direct path is the sum of the data processing delay plus the weapon to target transit time. For the command and control path, the total also includes the command and control processing delay. Spare time is the target dwell time minus this total time. Spare time is negative if the target moves before the weapon arrives and positive if the weapon arrives before the target moves. Lethality is the area under the spare time distribution where spare time is positive. It is obtained by subtracting the cumulative probability of zero spare time from unity. The baseline case assumes a coefficient of variation of one for both arrival rates and service rates at time one. Direct path lethality for the baseline case is 0.32. Baseline command and control lethality, with its longer delay, is 0.16. The coefficient of variation (standard deviation/mean) specifies both the variability in the workload arrival rate and the variability in the queue process service time. A random exponential process has a coefficient of variation of one. A deterministic process with a fixed service rate has a coefficient of variation of zero. A hyper-exponential process with more than random variability would have a coefficient of variation (CV) greater than one. As process variability is increased (CV increases), mean process queuing delay increases. Longer process delay leads to less spare time and a lower lethality. Table 1 shows lethality for both the direct and command and control paths. Squared coefficient of variation of both workload arrivals and queue service time increases from 0.5 to 4 over successive columns. All cases have the same mean workload (arrival rate) and mean capacity (service rate). Note the significant reduction in lethality with increases in variability. Thus workload and process variability statistics are important factors in the assessment of lethality.

CV sq.	Direct Path	C2 Path
0.5	.38	.23
1.0	.32	.16
1.5	.28	.14
2.0	.25	.12
4.0	.16	.10

Table 1. (U) Lethality Impact Of Service Time Variability Unclassified

(U) Dynamic Model

(U) In the dynamic model we vary the sensor workload and command and control queue service rate inputs with time. Table 2 shows results at dynamic times of 0, 1, and 2. The scenario represents an increase in sensor inputs from 1.2 per minute to 2.0 per minute. The command and control node performs other functions in addition to mobile targeting. Increases in these other functions reduce the service rate for mobile targets from 2 per minute to 1.5 per minute. The data processing queue is directly related to the sensor system and does not perform these other functions. Hence its service rate is not changed.

(U) The dynamic model also includes a throughput measure of effectiveness. Throughput is the reciprocal of queue utilization when utilization is greater than one. It is the proportion of messages that are not lost when workload exceeds capacity (arrival rate exceeds service rate). For utilization less than one no messages are lost and throughput is one. When the output of one queue feeds another the throughput of both is the product of the individual throughputs. In our case for the command and control path, the data processing queue output is the input to the command and control queue. Table 2 shows lethality and throughput for both the direct and command and control paths at each time step. Both arrival rate (Ar) and service rate (Sr) change with time. Note the resulting significant lethality and throughput changes with time.

			Lethality	
Time	**Ar**	**Sr**	**Direct**	**C2**
0	**1.2**	**1.25**	**.41**	**.30**
1	**1.6**	**1.75**	**.32**	**.16**
2	**2.0**	**2.25**	**.27**	**.08**
			Throughput	
Time	**Ar**	**Sr**	**Direct**	**C2**
0	**1.2**	**1.25**	**.98**	**.98**
1	**1.6**	**1.75**	**.95**	**.92**
2	**2.0**	**2.25**	**.83**	**.74**

Table 2. (U) Dynamic Model Lethality And Throughput Unclassified

Figure 5. shows the change in the cumulative throughput probability with time. Not only is throughput reduced with time but throughput uncertainty is increased. Later cumulative probability curves have a lower slope.

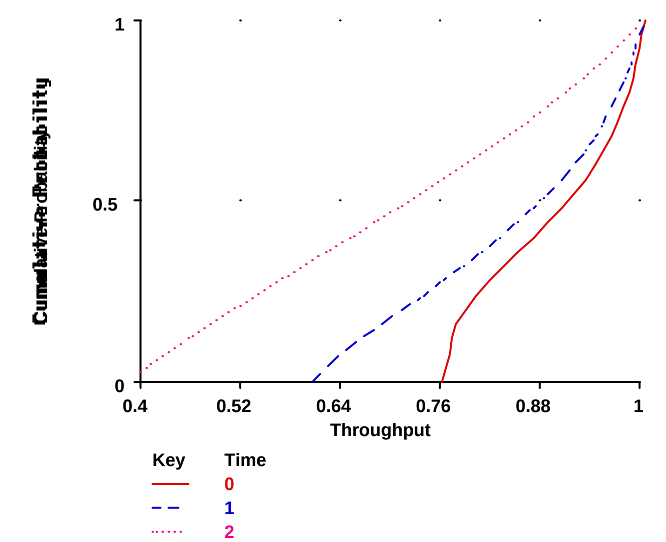

Figure 5. (U) Dynamic Throughput Cumulative Probability Unclassified

(U) **Conclusions and Extensions**

(U) The techniques illustrated have general application to uncertain process modeling. Processes can be modeled in greater detail by adding more component queues. In classified analyses we have also modeled communication links and satellites with queues. The concept of capturing a broad range of scenarios with a probability distribution also has wide application. The conversion between analytic queue fractiles and beta delay distributions provides a hybrid modeling capability. Analytic queuing is used to model the process and the beta method is used to summarize scenario parameters.

(U) We have implemented a library in Demos™ that allows one to develop models within a day. The deterministic analytic method gives the entire uncertainty distribution versus an uncertain estimate of the mean from discrete event simulation. This technique allows one to develop a dynamic probability surface. A time varying beta distribution can be fit to the time varying minimum, maximum, mean, and standard deviation statistics.

(U) Lethality can be extended to include execution planning and weapon effectiveness. In later work we have modeled the entire sensor to shooter chain including sensor performance, execution planning, air defense impact, and weapon effectiveness. Parametric scenarios are used to span the potential operating environment. The beta distribution is used to specify the distribution of target size, speed, contrast, cover and deception effectiveness, and temperature difference. Operating environments are characterized by theater size, terrain, and weather. Target classes with different dwell times employ different processing. Multiple target detections are used to track a mobile target and reduce errors in target identification. Inputs from multiple sensors also reduce identification errors.

(U) References

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